nificantly below the free-electron parabola. The small hump near the zone boundary predicted by the calculations of Gupta and Loucks is not resolved by the experiment. We observe however the hump predicted near $\theta = 6.5$ mrad. Thus the experiment supports the expectation of a pronounced contribution of the high-momentum components of the electron wave function where limitations in the convergence made the calculation somewhat uncertain.

We are grateful to J. C. Couterne of the Metallurgy Section of this Institute, who kindly grew and oriented the zirconium crystal for this experiment.

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Phonon Effects in the Far-Infrared Reflectivity of Superconducting Lead*

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Measurements of reflectivity of evaporated films of lead at 1.2 °K show phonon structure above the energy gap. The frequencies of the peaks in the spectrum agree with phonon frequencies in lead observed by tunneling and neutron spectroscopy.

Recently, Joyce and Richards[1] reported absorptivity measurements on normal and superconducting lead single crstals in the region 15–200 cm^{-1} showing structure they associate with phonon generation. In this note, we report on similar experiments on evaporated films of lead. We evaporated films of about 1000 Å in thickness on the inside of a stainless steel nonresonant cavity at room temperature. Measurements were carried out at 1.2 °K with a Michelson interferometer and we estimate that the radiation makes an average of 100 reflections with the walls before reaching the antimony doped germanium bolometer. In this way small changes in the reflectivity are easily observable. We calculate the ratio of the bolometer signal in the superconducting state to the signal in the normal state. A magnetic field of 1500–3000 G was sufficient to drive the film normal as shown by electrical measurements.

Figure 1 shows the ratio of the bolometer signal in the superconducting state to the signal in the normal state for a typical film at 1.2 °K. Simple analysis shows that the ordinate is approximately proportional to the difference in absorbance of cavity walls for the two states. We find the energy gap 2Δ at 20 cm^{-1} followed on the high-frequency side by phonon structure very similar to that reported by Joyce and Richards. It consists of two peaks, one at 55 cm^{-1} and the other at 87 cm^{-1}. Additional structure is evident at 42, 61, 75, and 102 cm^{-1}. At frequencies above 120 cm^{-1}, the curve rises smoothly and rapidly and levels off at 210 cm^{-1}.

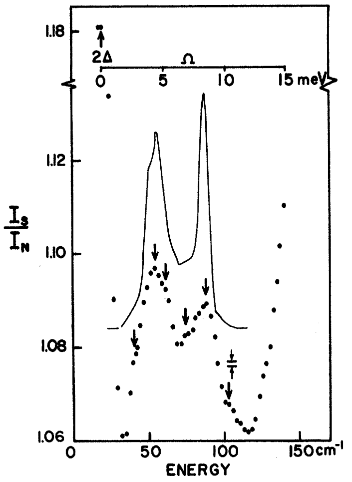

FIG. 1. Far-infrared reflectivity of superconducting lead films. Points show the change in absorbed power when a magnetic field is turned on and the film turns normal. Arrows denote discontinuities in the spectrum that are reproducible from sample to sample. The energy gap is at 2Δ and the meV scale is measured from this point. The solid curve is $\alpha^2F(\Omega)$ obtained from tunneling measurements in Ref. 2. The resolution along the energy scale is 5 cm^{-1}.

The magnitude of this relative increase in absorbance at high frequencies in the normal state is stronger in our films than in the single-crystal measurements of Joyce and Richards. In our case the energy to the bolometer changes by 12% in the gap region and by 20% at 210 cm^{-1}. The relative strength of the peaks at 55 and 87 cm^{-1} changes with the age of the film (stored at 77 °K). This effect is related to an increase of reflectivity at low frequency relative to high frequency and is more marked in the superconducting state. As a result, after several days the 89 cm^{-1} peak appears to weaken.

Joyce and Richards propose a mechanism for the structure where the radiation generates electronic excitations of energy 2Δ plus a phonon of energy $\hbar\Omega$. They found however a disagreement of some 6 cm^{-1} between the $\hbar\Omega+2\Delta$ expected and their observed structure. We find no such disagreement in our measurements. From tunneling data[2] the main peaks in $\alpha^2F(\Omega)$, the quantity proposed to describe the process, occur at 56 and 87.5 cm^{-1} in excellent agreement with our peaks at 55 and 87 cm^{-1}. The additional structure we observe can be correlated with various phonon features seen in tunneling and neutron scattering but our signal-to-noise ratio (1000) does not permit an unambiguous identification of this structure. The discontinuity at 42 cm^{-1}, however, does not correspond to any known phonon singularity in lead but agrees with tunneling data. Our experimental method is very similar to that of previous investigators,[3] but the phonon structure described here is at somewhat higher frequency than the region of interest to those workers, and requires a high signal-to-noise ratio for identification.

In conclusion it appears that the phonon generation process found by Joyce and Richards is clearly observable in lead films and provides an alternative to tunneling measurements.

We would like to thank J. P. Carbotte for encouraging us to do this work.

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Semiclassical Analysis of Field Emission through Atoms Adsorbed on Metal Surfaces*

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INTRODUCTION

Recent experiments on the field emission of electrons through atoms adsorbed on metal surfaces[1-3] have demonstrated the importance of a resonance tunneling mechanism.[1-5]

One simple theoretical approach employs a semiclassical (WKBJ) analysis of the one-dimensional potential shown in Fig. 1.[3-5] This is a problem that has been investigated by semiclassical techniques, not only in solid-state and surface physics,[3-7] but also in chemical physics when the potential energy barrier for a reactive collision contains a well,[8,9] and in nuclear physics when the fission barrier for heavy nuclei possesses two maxima.[10,11]

The purpose of this note is first to draw attention to this body of work on resonance tunneling in other branches of physics, and second to consider certain interesting features of resonance tunneling within the semiclassical approximation. In particular, for energies lying well below the barrier maxima, the transmission coefficient is expressed explicitly in terms of the resonance energies and widths of the quasistationary states that exist within the well of the potential [see Eq. (5) below]. This is in accord with the results of Refs. 2–5. The calculations of Ref. 9 are used.

TRANSMISSION COEFFICIENT

An expression for the transmission coefficient D valid for energies E lying above or below the barrier maxima $V(b_1)$ and $V(b_2)$ is considered first, and then specialized to the case when the energy